

# **R\_FOFO snake channel for 6D muon cooling**

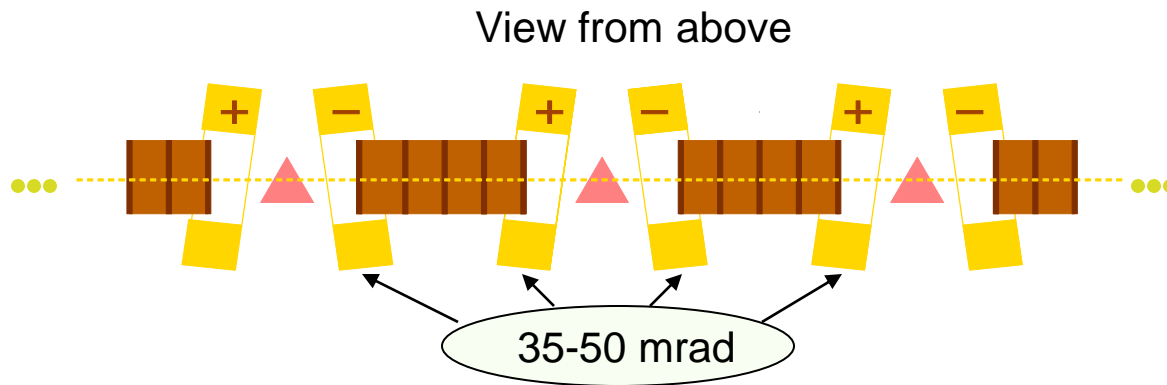
(“R” can be interpreted as “rectilinear”)

V. Balbekov, MAP Friday Meeting 02/01/2013 (edited 06/01/13)

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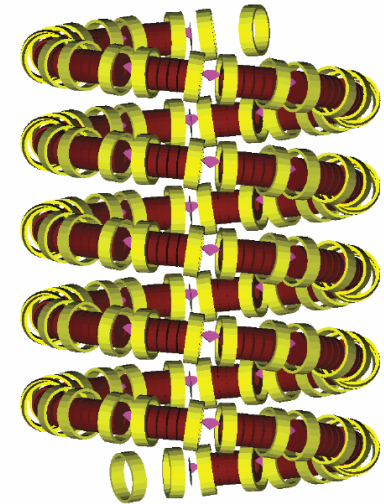
Thanks to Yuri for the idea to use RFOFO cells for helical or snake 6D cooling channel as well as for numerous discussions and advises.

# Schematic of the R\_FOFO snake

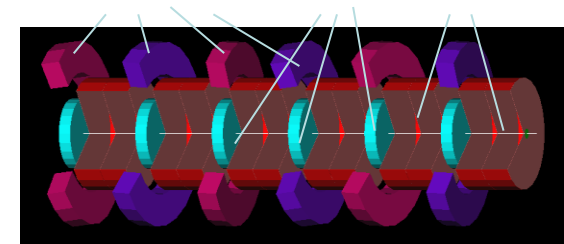


- The R\_FOFO “snake” is actually a rectilinear channel with tilted alternating solenoids and wedge absorbers.
- It is similar in appearance to the helical FOFO snake. However, use of wedge absorbers instead of planar ones essentially changes features and applicability.
- Substantially, the R\_FOFO is closer to the Guggenheim channel because:
  - Both of them can be composed of identical cells being different only in arrangement of the parts;
  - They have almost the same characteristics (acceptance, ultimate beam emittance, transmission, etc.);
- At the same time, the R\_FOFO snake is significantly simpler in construction.

Guggenheim



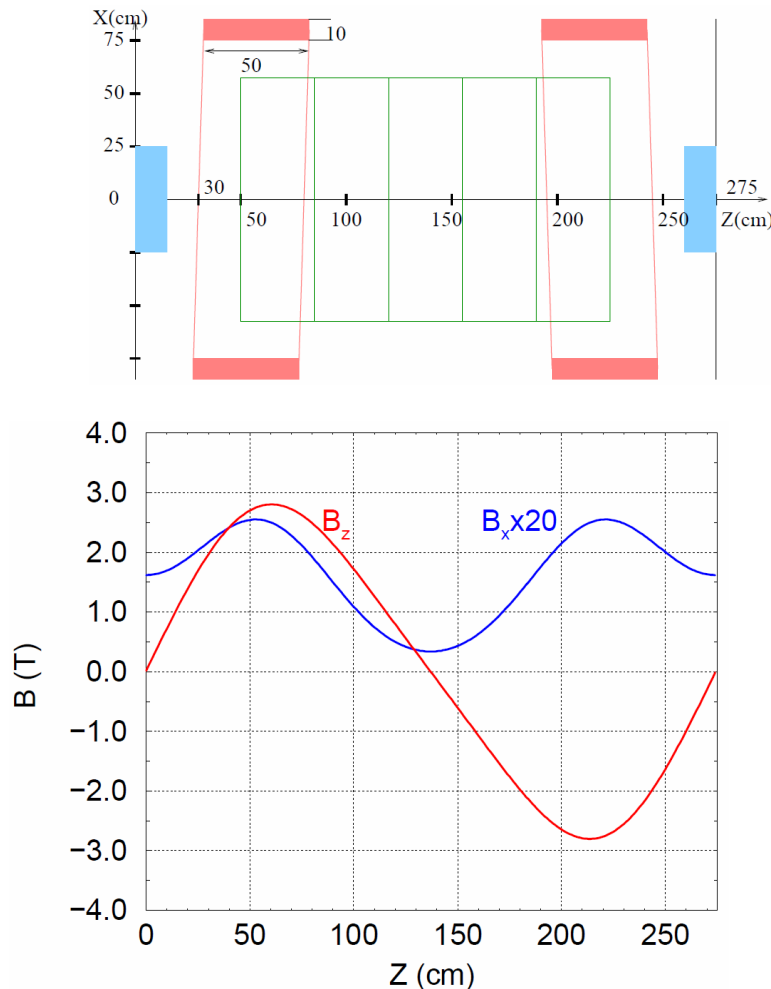
Helical FOFO snake



# R\_FOFO snake with 2.75 m cells

(Guggenheim cells by P. Snopok, G. Hanson, and A. Klier, IJMPA 24-5, 987, 2009)

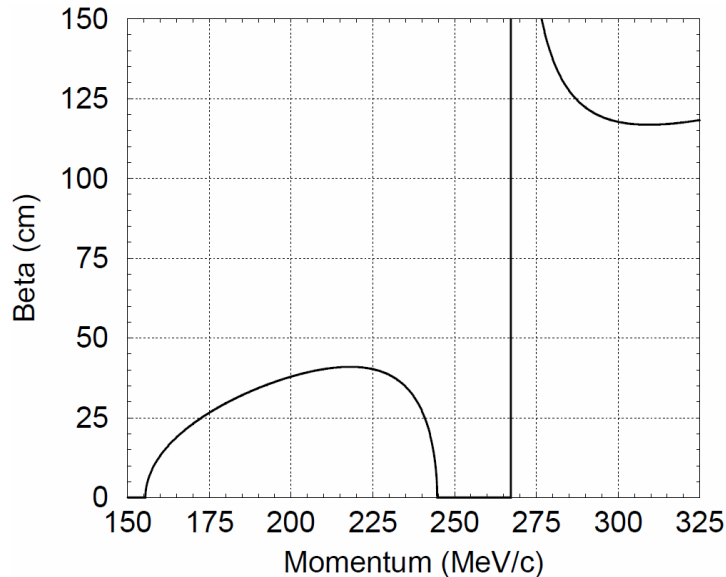
Cell schematic and field at X=Y=0



## List of parameters

Period length	275 cm
Solenoids inclination	35 mrad
Maximal field strength on axis:	
longitudinal	2.80 T
horizontal	0.13 T
Maximal field strength in coil	7.22 T
Current density	102 A/mm <sup>2</sup>
Reference momentum	210 MeV/c
Accelerating frequency	200 MHz
Accelerating gradient	13.5 MV/m
Synchronous phase	25°
Absorber LH <sub>2</sub>	
thickness on axis	32 cm
opening angle	78°

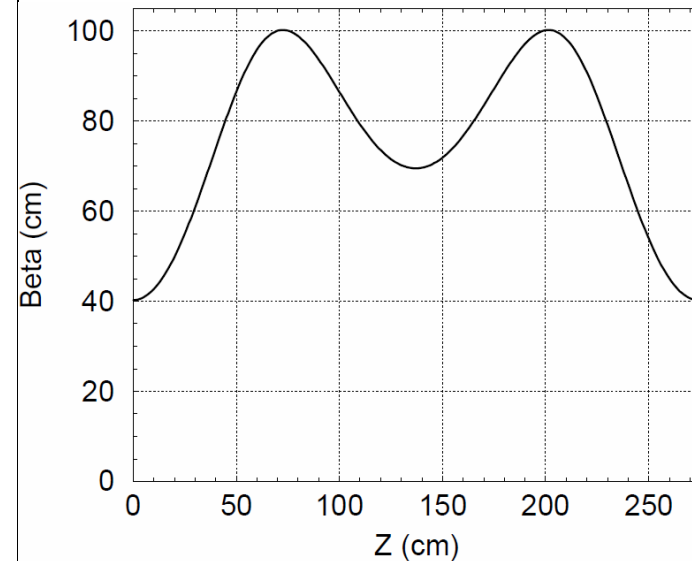
# Beta-function



Beta-function at the absorber center against muon momentum.

Working zone  $155 \text{ MeV/c} < P < 245 \text{ MeV/c}$  is bounded by integer and half-integer resonances.

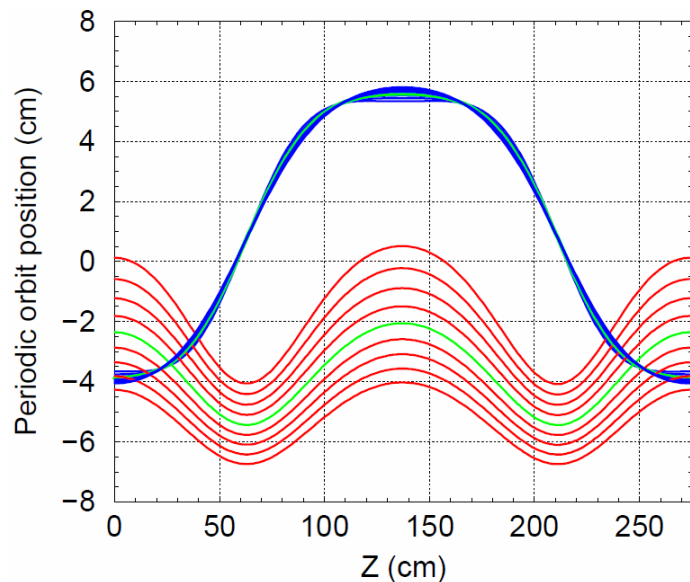
Phase advance is about  $3\pi/2$  per cell at the center of the working zone and it is less of  $\pi$  at  $P > 275 \text{ MeV/c}$



Beta-function at 210 MeV/c against longitudinal coordinate starting from the absorber center.

Minimal beta is 40 cm

# Periodic orbit and dispersion

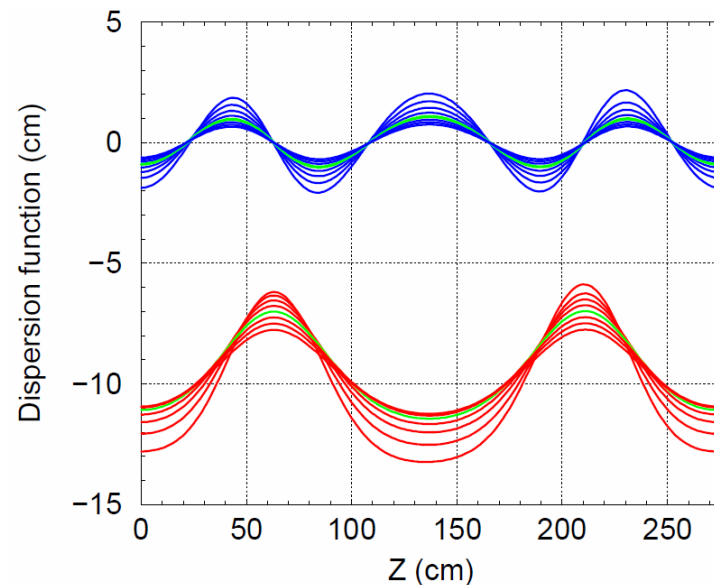


Several periodic orbits are plotted against longitudinal coordinate starting from the absorber center.

Muon momenta are 170-250 MeV/c, step 10 MeV/c.

Blue – horizontal, red – vertical.

P=210 MeV/c is marked by green.



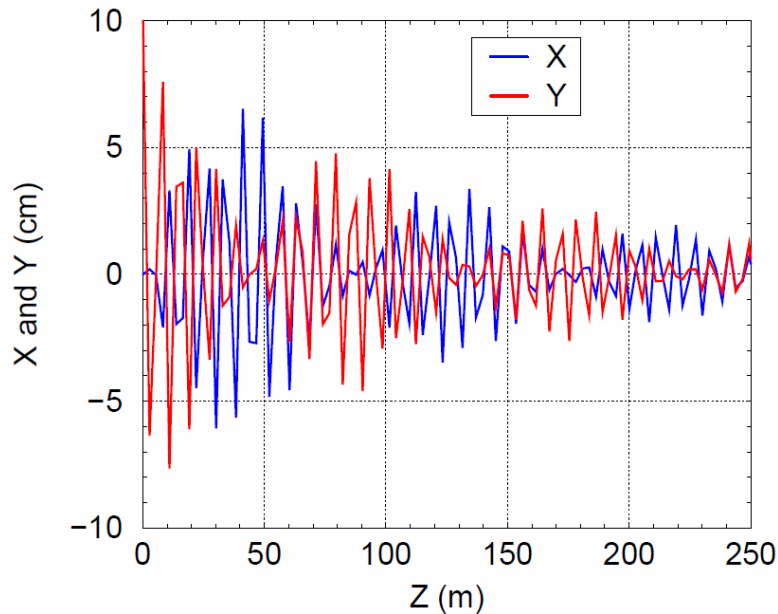
Dispersion function against longitudinal coordinate starting from absorber center. Muon momenta are 170-240 MeV/c, step 10 MeV/c.

Blue – horizontal, red – vertical.

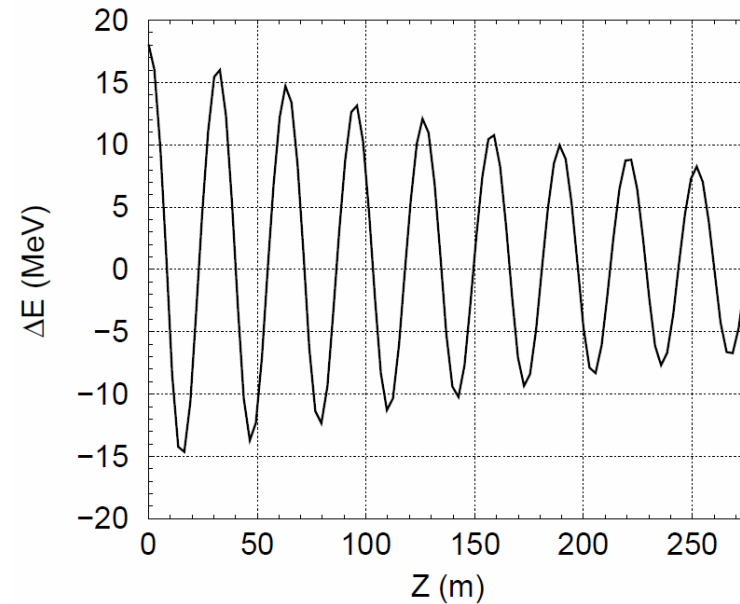
P=210 MeV/c is marked by green.

$D_x = -11$  cm (Guggenheim – 8 cm)

# Tracking simulation (no stochastic effects)

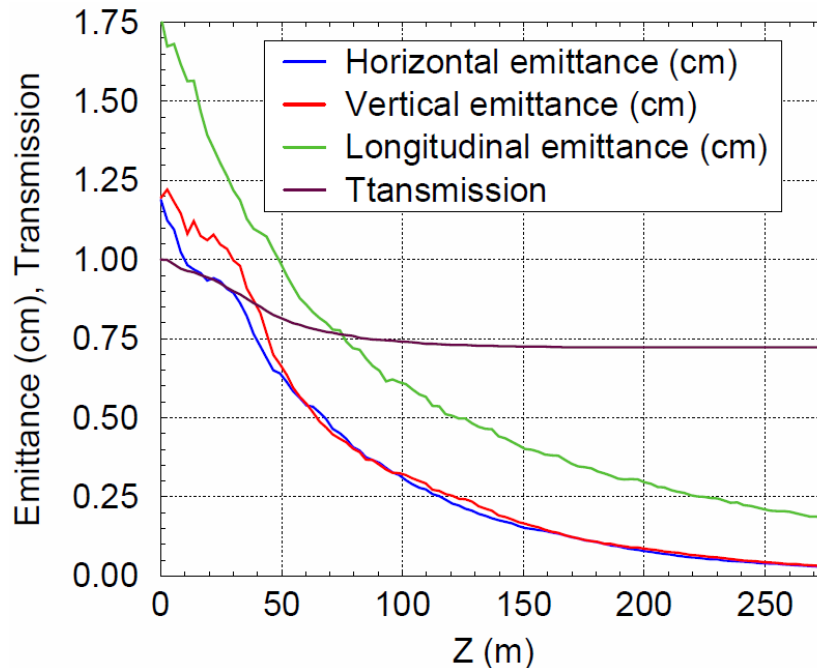


Betatron oscillations of a particle with momentum 210 MeV/c



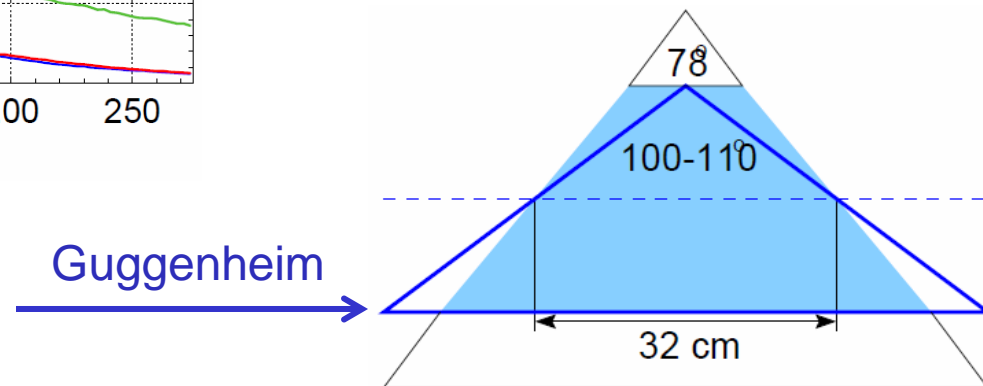
Synchrotron oscillations of a particle with 200 MHz RF and 210 MeV/c reference momentum.

# Cooling simulation without stochastic effects



The parameters are chosen to get about equal cooling rates in all direction. Can be changed (optimized) by variation of the solenoid tilt and/or wedge absorber angle.

Below:  $\text{LH}_2$  absorber shape  
(R\_FOFO snake (filled) and Guggenheim)



# Cooling simulation with stochastic effects

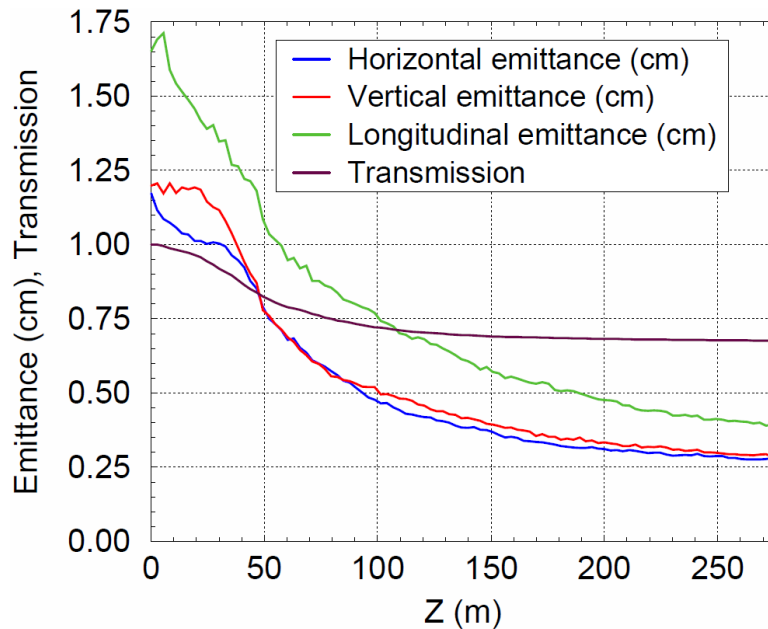
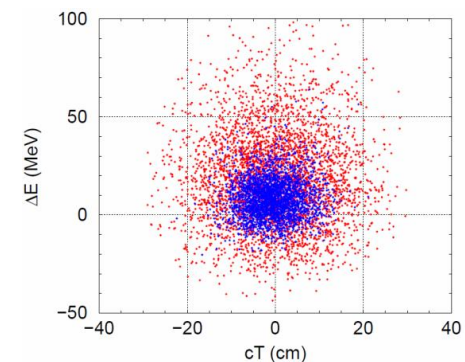
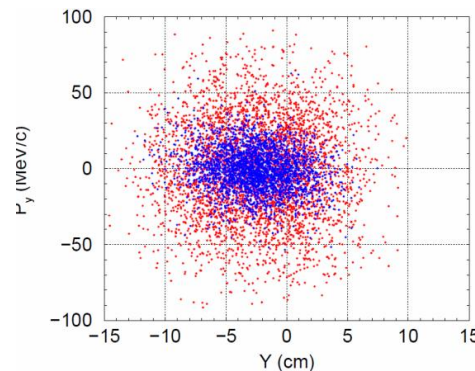
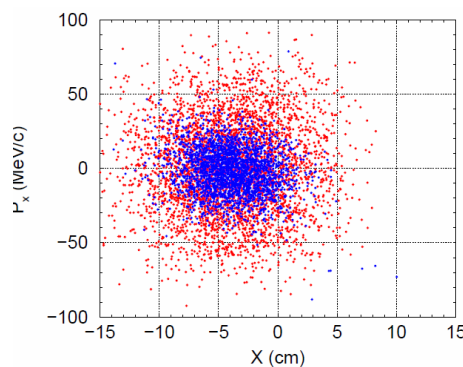


Table: beam parameters after 200 m  
(R\_FOFO compared with Guggenheim)

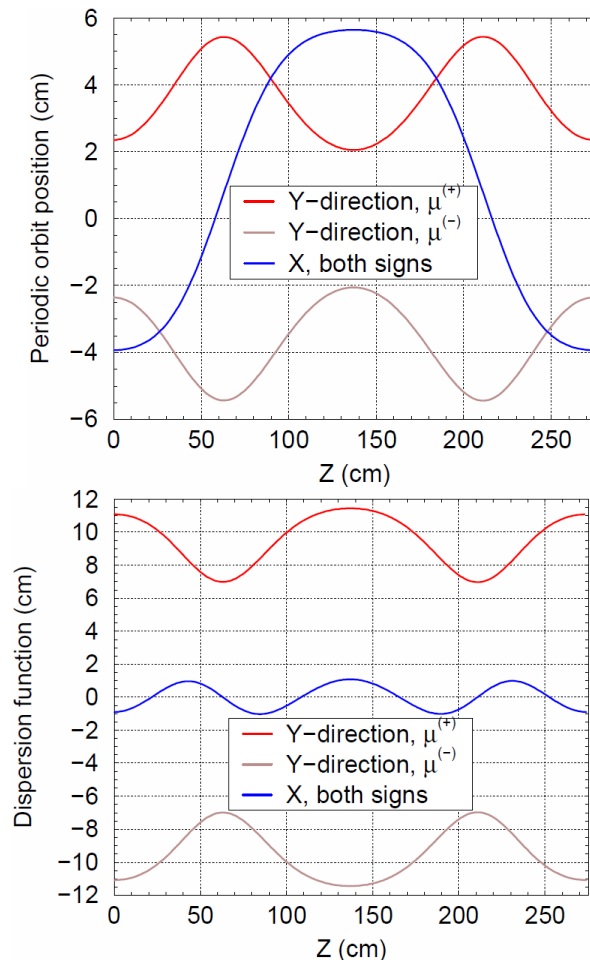
	R_FOFO	Gug
Trans. emit. (mm)	3.2	3.7
Long. emit. (mm)	4.8	6.1
Transmission with decay (%)	58	62

Below: phase space in the beginning and after 275 cm:  $X$ - $P_x$ ,  $Y$ - $P_y$ ,  $cT$ - $\Delta E$  (cm, MeV)



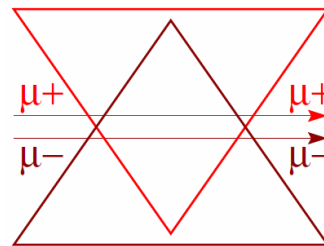


# Is it possible to cool muons of both signs together?



Different absorbers needed for  $\mu$

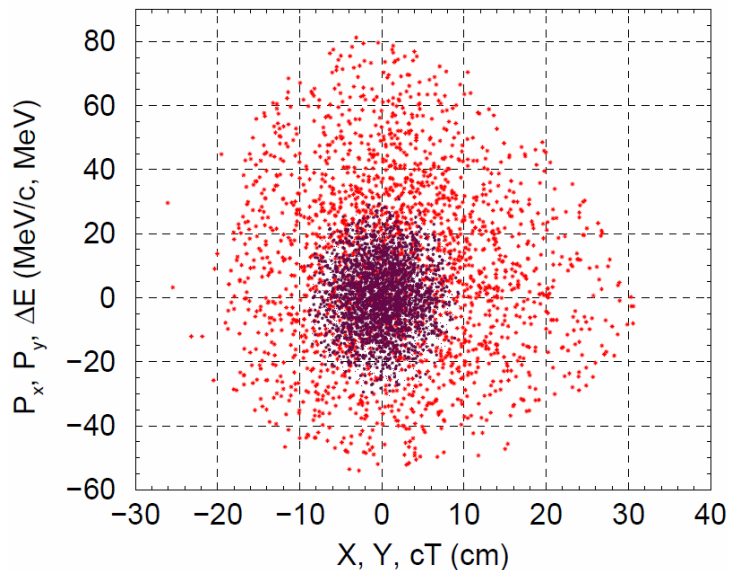
- Horizontal periodic orbit and dispersion function do not depend on muon sign being directed by longitudinal solenoid field.
- Vertical periodic orbit and dispersion function are placed symmetrically in accordance with muon sign.
- Therefore opposite located and shaped wedge absorbers are needed for positive/negative muons.
- Consequently, the R\_FOFO snake is unfit for cooling of  $\mu$  simultaneously.
- Planar absorbers would be needed for this like Yuri helical FOFO snake which is suitable for  $\mu$
- But HFOFO incompatible with RFOFO idea because the planar absorbers are placed where  $\beta$ -function is maximal.



Challenge: How to include planar absorbers into a channel with strongly modulated beta-function for 6D cooling (???)

# Is R\_FOFO compatible with Front-End system (325 MHz)

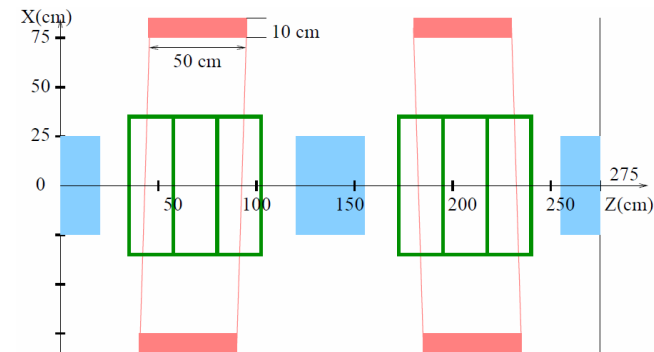
Front-end beam distribution by D. Neuffer



**Red** – longitudinal distribution after collection of all particles of a bunch in a single 325 MHz bunch. The separatrix is full, **the bunch rms emittance is 2.7 cm**.

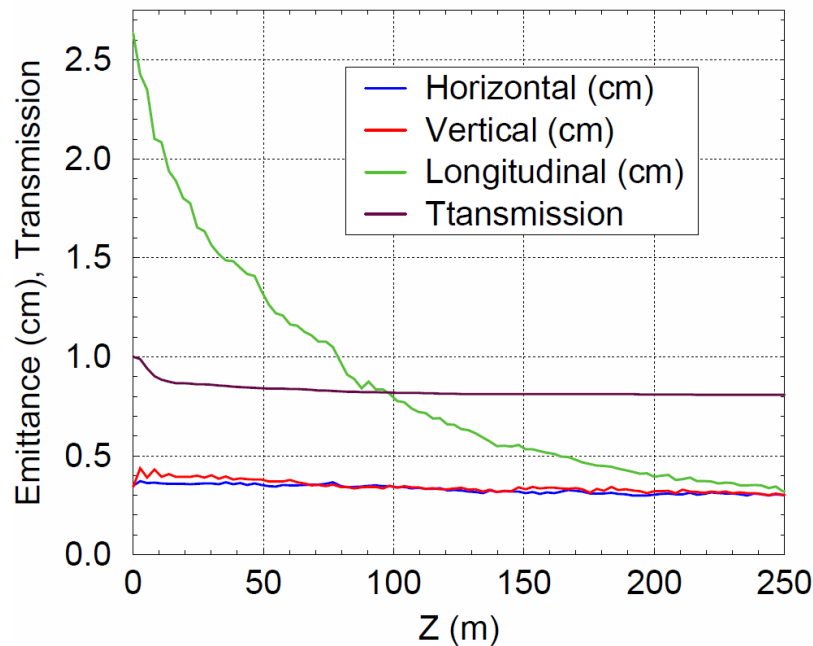
**Maroon** – any transverse distribution:  $X-P_x$ ,  $X-P_y$ , etc. **Emittance 0.34 cm**  
It is a collimated distribution with LiH absorber

2.75 m cell is slightly modified



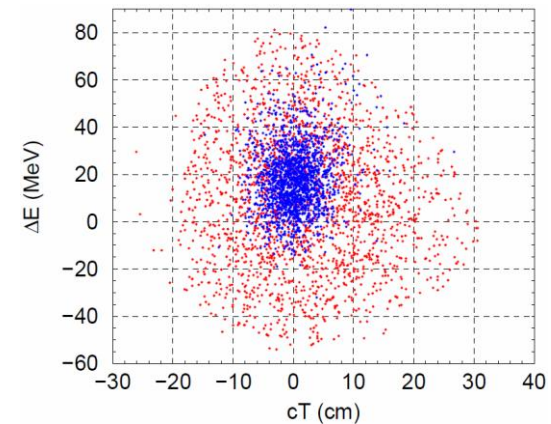
Accelerating frequency	200 → 325 MHz
Reference momentum	210 → 245 MeV/c
Accelerating gradient	13.5 → 20 MV/m
Synchronous phase	25° → 30°
Absorber thickness	32 cm → 34 cm
Absorber angle	78° → 80°
Current density	102 → 119 A/mm <sup>2</sup>
Long. field on axis	2.80 T → 3.40 T
Hor. Field on axis	0.13 T → 0.15 T
Maximal field in coil	7.22 T → 8.37 T

# Cooling simulation with 2.75 m/325 MHz R\_FOFO



Longitudinal cooling by the R\_FOFO snake after front-end.

Final emittances 0.3 cm in all directions, transmission 80%.

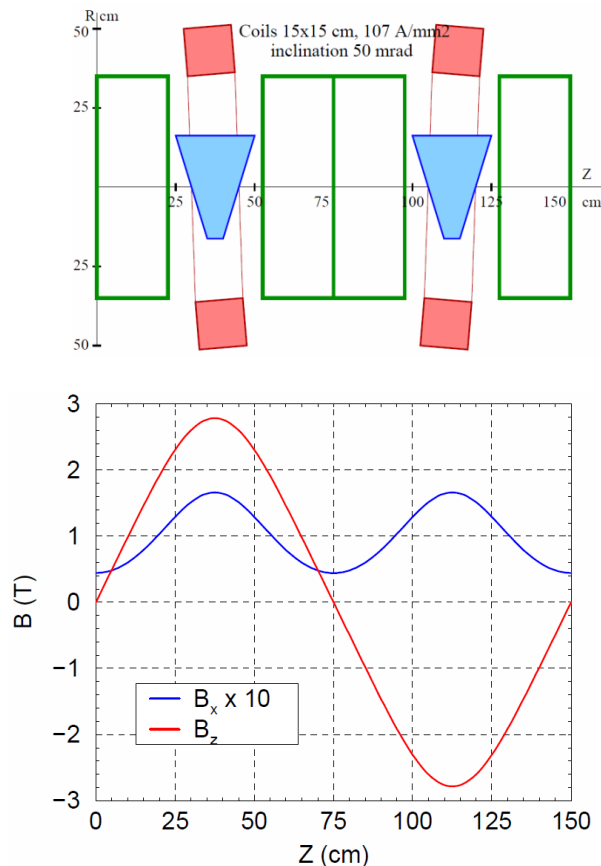


Longitudinal phase space before and after the cooling.

# Is it possible to convert the Front-End 4D to 6D cooling channel?

## Front-End cell from David is used as basis

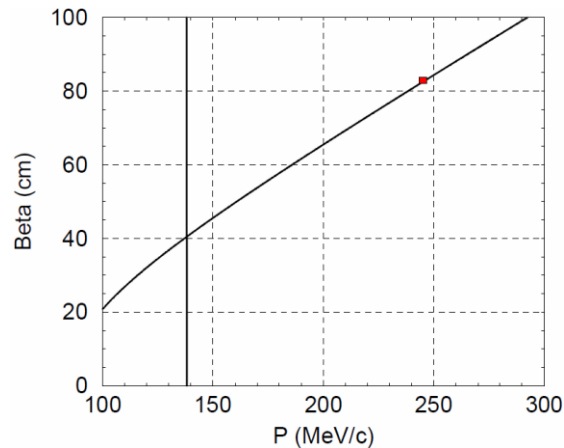
The solenoids are inclined by 50 mrad, and the absorbers are wedge-shaped



## List of parameters

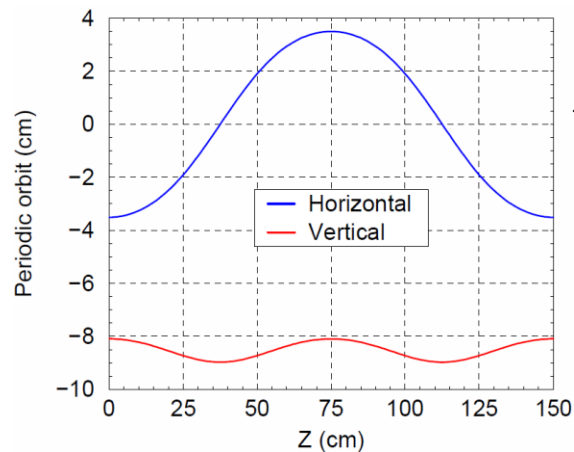
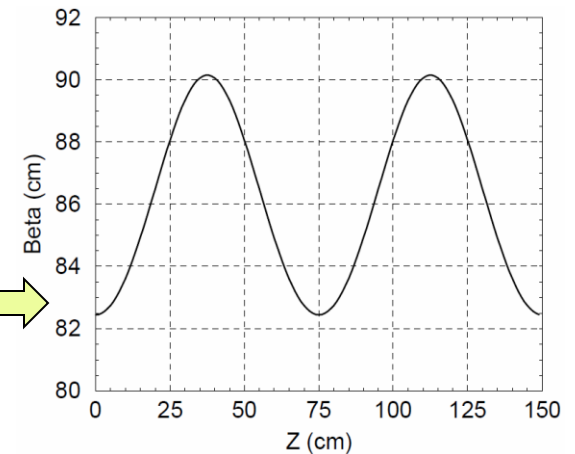
Period length	150 cm
Maximal field strength on axis:	
longitudinal	2.78 T
horizontal	0.17 T
Maximal field strength in coil	7.25 T
Current density	107 A/mm <sup>2</sup>
Solenoids inclination	50 mrad
Reference momentum	245 MeV/c
Accelerating frequency	325 MHz
Accelerating gradient	25 MV/m
Synchronous phase	30°
Absorber LH <sub>2</sub>	
thickness on axis	17 cm
opening angle	38°

# Modified FE channel: beta-function, periodic orbit, dispersion



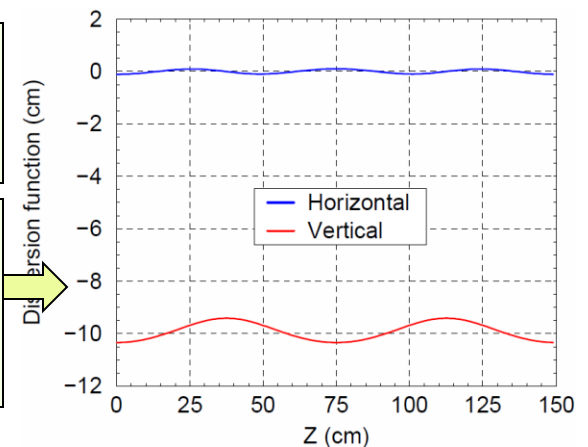
Beta-function vs momentum  
Weak parametric resonance  
appears at  $P=138$  MeV/c

Beta-function vs longitudinal  
coordinate Average beta is  
86 cm, modulation 4%

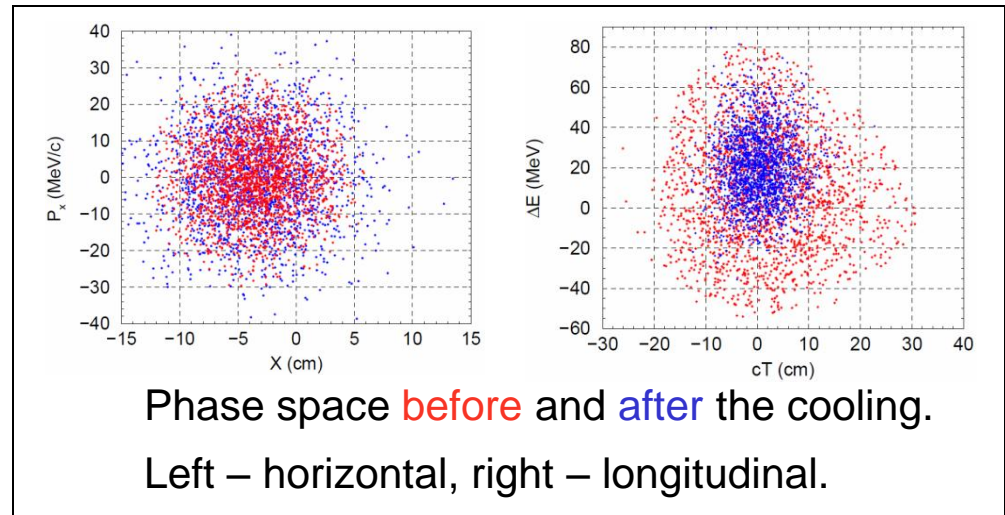
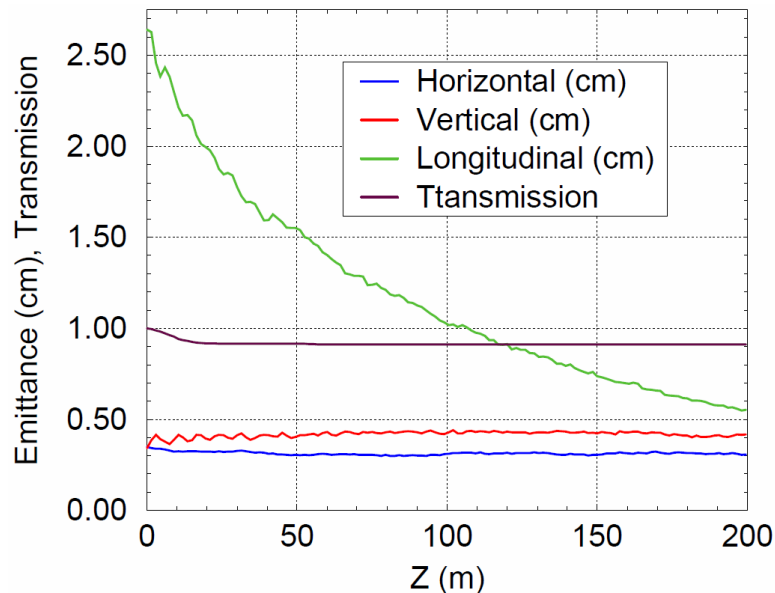


Periodic orbit at 245 MeV/c.  
Horizontal orbit almost does  
not depend on momentum

Dispersion function at  
245 MeV/c. Horizontal  
dispersion is  $\approx 0$ , vertical one  
is about -10 cm everywhere.



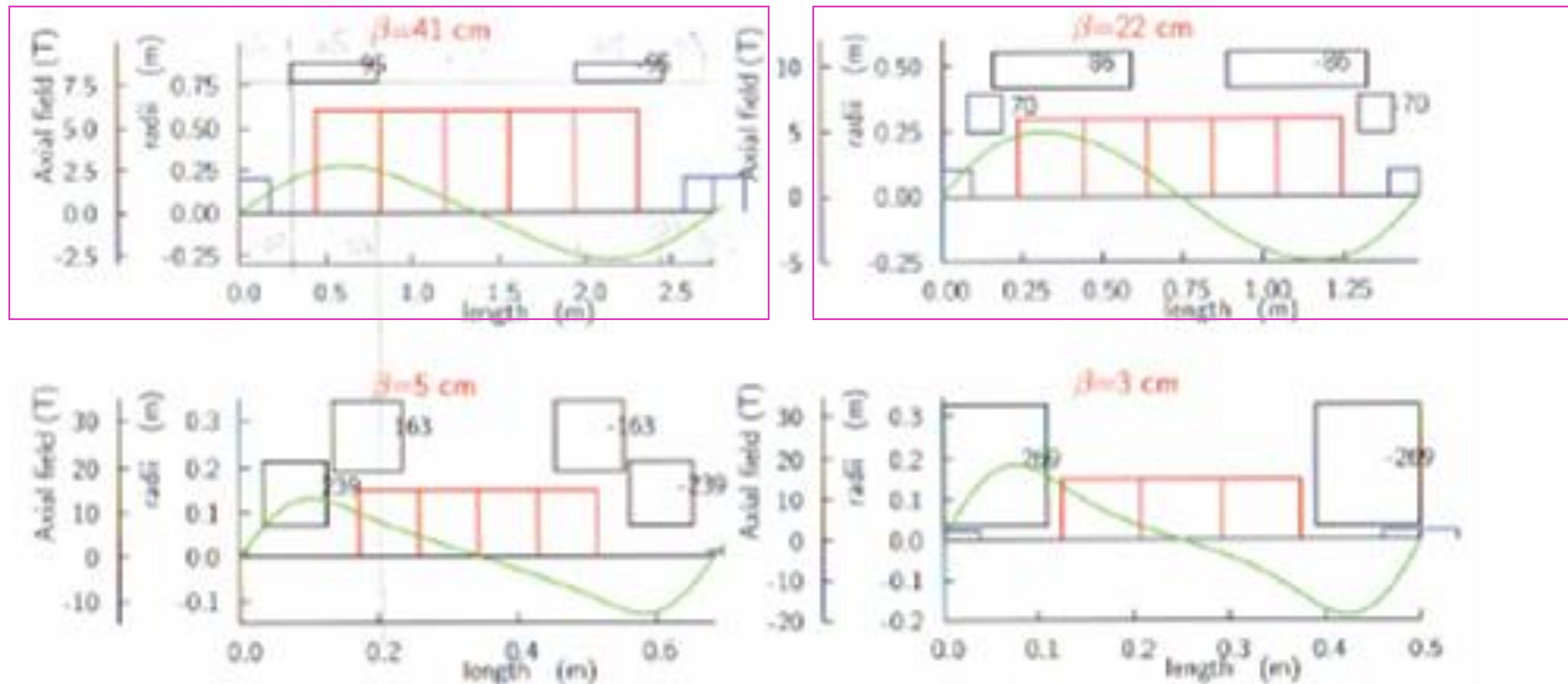
# Cooling by modified FE channel with tilted solenoids



- Tilting of solenoids + wedge absorbers can transform 4D to 6D cooling channel.
- It provides good longitudinal cooling and 90% transmission.
- Modest growth of transverse emittance results from decrease of transverse decrement due to emittance exchange.
- In principle, it allows to incorporate 6D cooling into front-end channel
- It is unsuitable for simultaneous cooling of  $\mu$  but can be useful for v-factory
- Alexahin's HFOFO snake looks as a better choice for  $\mu$ , because beta-function is not perceived to be very small in the phase rotation – precooling sections.

## B.Palmer & Rick Fernow, MAP Friday Meeting 10/26/12

### Examples of RFOFO Lattices

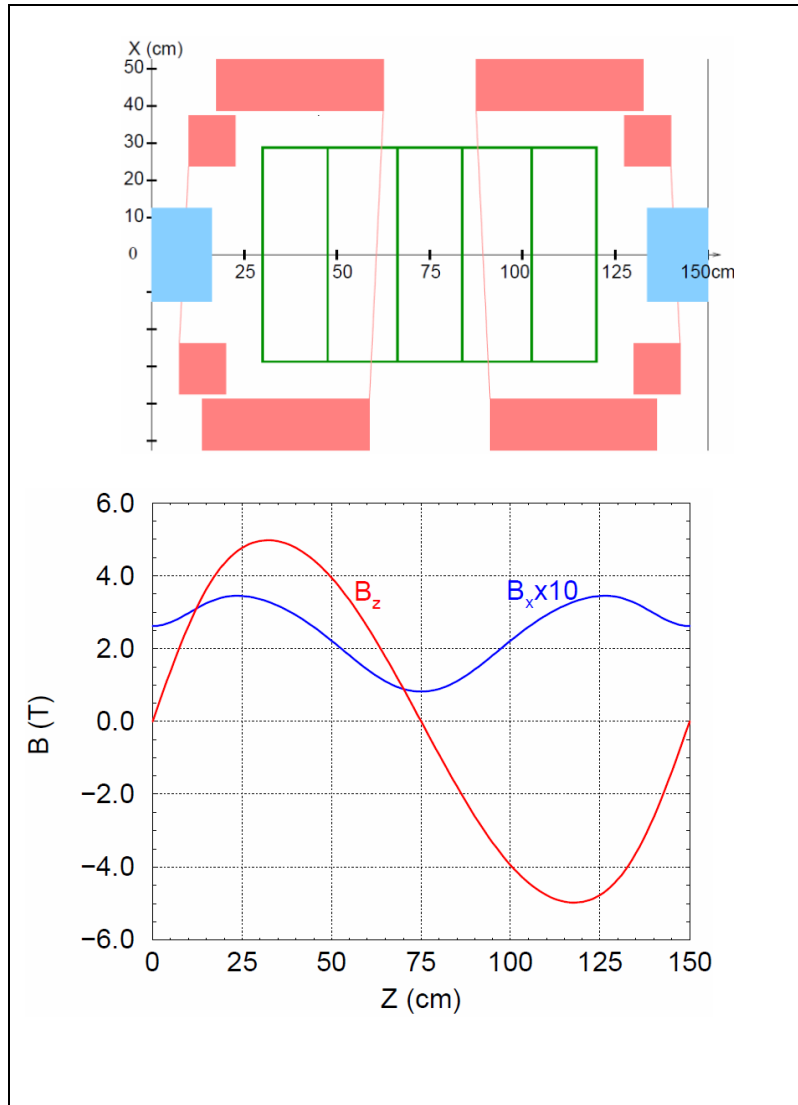


- Current densities are very high in low beta examples
- Fields change rapidly  $\rightarrow$  field have strong radial components



# Cooling channel of less beta: lattice, field, other parameters

Cell schematic and field at X=Y=0

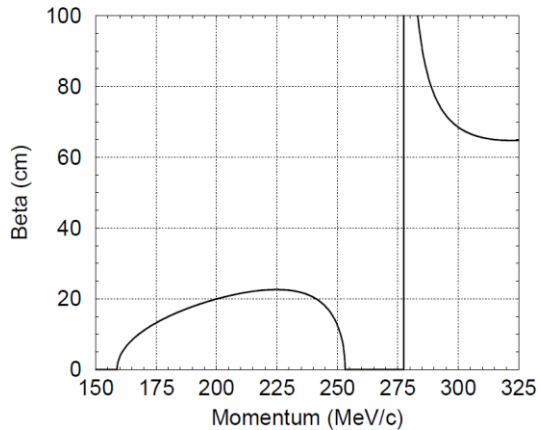


## List of parameters

Period length	150 cm
Maximal field strength on axis:	
longitudinal	4.98 T
horizontal	0.34 T
Maximal field strength in coil	7.71 T
Current density	69/94 A/mm <sup>2</sup>
Solenoids tilt	50 mrad
Reference momentum	210 MeV/c
Accelerating frequency	400 MHz
Accelerating gradient	22 MV/m
Synchronous phase	30°
Absorber LH <sub>2</sub>	
thickness on axis	32 cm
opening angle	78°

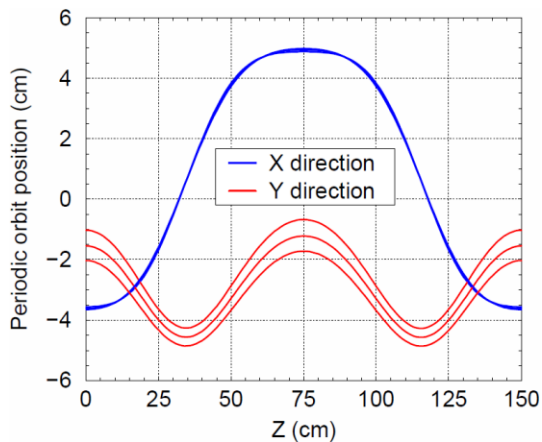
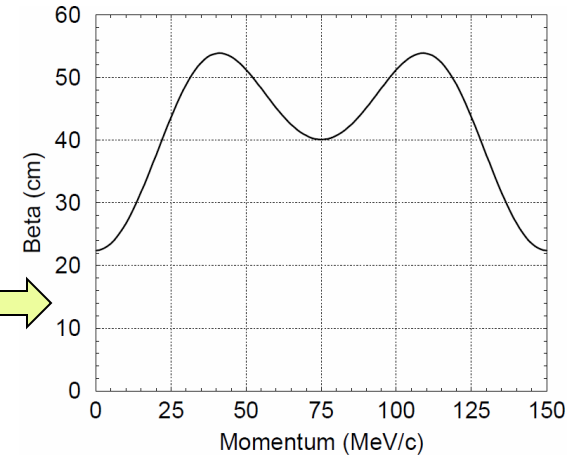


# Beta-function, periodic orbit, dispersion



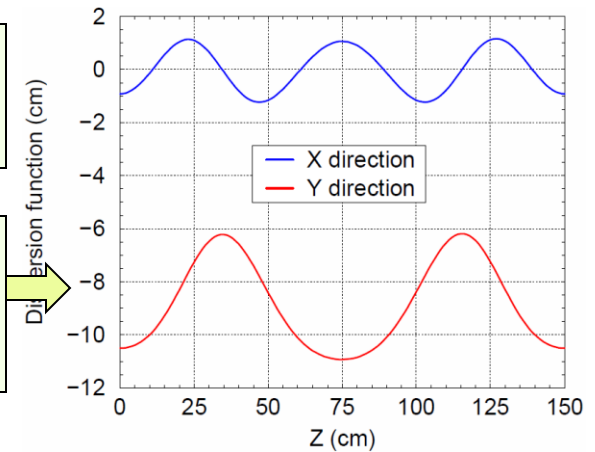
Beta-function vs muon momentum. Working zone is 165-255 MeV/c,

Beta-function vs longitudinal coordinate. Minimal beta is 22 cm

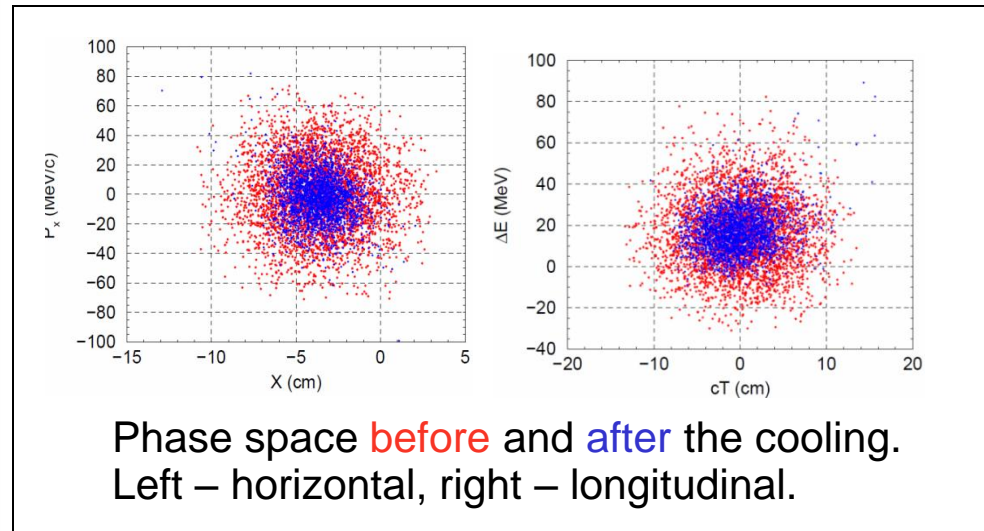
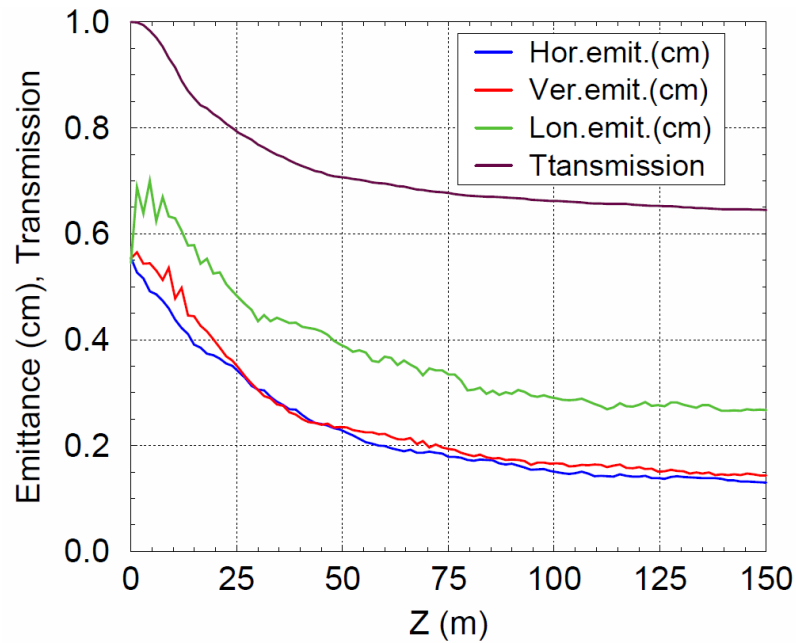


Periodic orbit at 200-210-220 MeV/c.

Dispersion function at 210 MeV/c. Vertical dispersion at the absorber is -10.5 cm



# Cooling simulation



Initial emittance 0.6 cm is accepted at all directions as the phase rotation – precooling channel can provide (hopefully)

With the same absorber as previously (32 cm,  $78^\circ$ ), achievable transverse emittance is about 1.5 mm which is coming with beta-function 22 cm.

Without decay, transmission is 65% at 400 MHz RF (but it is 79% at 200 MHz).

Violation of longitudinal motion due to dependence of particle time of flight on betatron amplitude is the main cause of the loss.

# Summary

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- R\_FOFO snake channel with tilted solenoids is usable for 6D cooling
- Essentially, any FOFO or RFOFO 4D-cooling channel can be converted into 6D-channel by inclination of solenoids (typically on 30-60 mrad) and use of wedge absorbers
- The R\_FOFO is easy adjustable with front-end (phase rotation) channel for v-factory. However, because of wedge absorbers, it is unsuitable for simultaneous cooling of  $\mu$
- As it is simulated, transverse emittance 1.8 mm and longitudinal one 2.4 mm are achievable with magnetic field 5 T in axis, 7.7 T in the coil.
- Emittance less of 1 mm looks to be a realistic goal with magnetic field 10-12 T in coils.
- However, particles loss due to dependence of flying time on betatron amplitude is a serious constraining factor for transmission. Lower accelerating frequency is preferable from this point of view.